Data Fusion for Hybrid and Autonomous Time-of-Flight Positioning

Aymen Fakhreddine
IMDEA Networks Institute
& University Carlos III of Madrid
Madrid, Spain
aymen.fakhreddine@imdea.org

Domenico Giustiniano
IMDEA Networks Institute
Madrid, Spain
domenico.giustiniano@imdea.org

Vincent Lenders
armasuisse
Thun, Switzerland
vincent.lenders@armasuisse.ch

ABSTRACT

Existing mobile devices such as smartphones rely on a multi-radio access technology (RAT) architecture to provide pervasive location information in various environmental contexts as the user is moving. Yet, existing architectures consider the different localization technologies as *monolithic* entities and choose the final navigation position from the RAT that is expected to provide the highest accuracy. In contrast, we propose to *fuse timing range measurements* of diverse radio technologies in order to circumvent the limitations of the individual radio access technologies. We take a first step in this direction and propose to fuse timing measurements of satellite navigation systems and WiFi networks. We introduce different novel methods such as a data fuser, an estimator of WiFi ToF distance and a geometrical-statistical approach to best fuse the set of ranges in presence of a rich set of measurements. Experimental results show that our solution allows the mobile device to efficiently position itself in diverse challenging scenarios.

1 INTRODUCTION

The proliferation of handheld devices and the pressing needs of location-based services call for precise and accurate mobile positioning. Despite the large investments and efforts in academic and industrial communities, a pin-point solution is still far from reality [1]. Mobile devices mainly rely on the Global Positioning System (GPS) to position themselves, known to perform poorly in dense urban areas and indoor environments. In order to ensure interoperability among the technologies used indoors and outdoors, a pervasive positioning system should still rely on GPS, yet complement it with other technologies that are integrated in commercial mobile devices.

Existing mobile devices such as smartphones commonly rely on multi-RAT (Radio Access Technology). One critical aspect for the localization problem is that the various technologies operate as *monolithic radios* entities. Here the principle is to compute the final positioning selecting the radio technology that is foreseen to provide the highest accuracy [2, 3].

In this work, we pose the question of how a device could enrich the set of high-quality ranging measurements for those scenarios with limited number of GPS satellites (*no position fix*), or in presence of GPS pseudoranges largely affected by multipath (a rich set of GPS measurements, yet *low accuracy of the position*), such as indoors. We propose to fuse range measurements of diverse radio technologies in order to circumvent the limitations of the individual radio access technologies and improve the overall localization accuracy. The concept of fusion has been a key to achieve reliable position fixes in outdoors with GPS measurements and inertial sensors [4]. In contrast, the literature has only scratched the surface of the problem of how to model and exploit raw ranging measurements from heterogeneous technologies [5].

In this work, we take a first step in this direction and we propose to fuse timing information (Time-of-Flight (ToF)) extracted from GPS and WiFi technologies. The latter technology has been selected due to its large availability in indoor areas and widespread usage in mobile devices. Yet, new problems emerge in presence of new and diverse ranges:

Heterogeneous sources of noise. Ranging inputs are made available from diverse technologies for the final position calculation and are subjected to heterogeneous sources of noise that must be accurately modeled and tackled, otherwise making the fusion ineffective.

Accurate ToF WiFi ranges. Ranging in GPS is well understood. Fusing adds new ranges from WiFi and how to get accurate WiFi ranges needs to be understood as well.

What ranges measurements to use. A large number of multi-RAT ranges could be available and we question how to select these ranges to achieve both accurate and pervasive position estimates. This diversity can be detrimental in the presence of one or more bad quality measurements which may bias the final position.

Autonomous solution. The computation of WiFi ranges must not be hindered by chipset-specific problems and should not require environmental calibration.

For this new architecture, our contributions are listed as follows:

- We propose a data fuser operating in two phases, which properly models the process and measurement noise. A filter automatically learns the bias of WiFi chipsets of new APs in range.
- We reliably estimate ToF ranges without performing any calibration of the environment.
- We consider diversity as an asset, detecting and identifying faulty measurements to discard them, and propose a novel geometrical-statistical approach that is more suitable to hybrid localization systems than methods presented in the literature for single technologies.
- We evaluate the system in an indoor testbed using commodity hardware. We collect raw ToF ranging measurements from both technologies and demonstrate that our system outperforms other strategies in diverse settings.

2 MOTIVATION

As of today, satellite navigation and network communication have operated according to isolated structures, with limited efforts to

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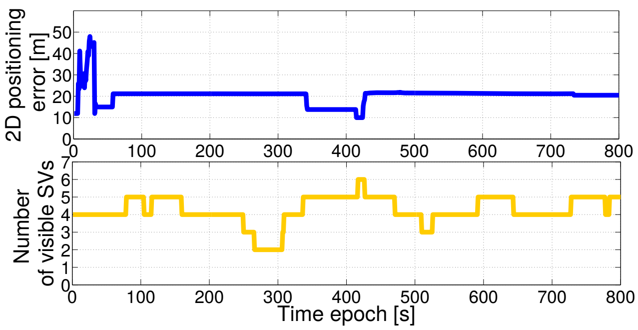

Figure 1: Skyhook's accuracy and number of visible satellites in range.

introduce an architecture that integrates signals of terrestrial communication and satellite navigation signals at a high granularity [5]. The most successful approach so far to integrate satellite navigation and data network communication is Assisted-GPS (AGPS), as it greatly helps to reduce the time-to-first fix (TTFF) by sending ephemeris and almanac data through communication networks. However, it does not help to increase the coverage nor the accuracy of the navigation service [3]. There has also been very little attention to the problem of understanding how to merge timing measurements for ranges coming from different technologies. [6] studies the ambiguity in the position using both GPS and WiFi range measurements, yet it does not try to outperform standalone GPS's accuracy.

In order to explore the problem space, we undertake a measurement to understand how a commercial hybrid solution works in a constrained environment.

Hybrid solution in the market. We test the accuracy of the Skyhook wireless mobile location service using an Android smartphone in an university campus. Skyhook is a hybrid positioning system leveraging optimal combinations of WiFi, GPS, and Cellular [1]. The test is performed indoor. While testing it, we deactivate the communication through cellular networks to make sure we use only GPS and WiFi. Figure 1 (bottom) shows the number of visible GPS satellites (SVs). We show the accuracy of Skyhook in Fig. 1 (top). The best accuracy is achieved in the short time when 6 satellites are available. As GPS requires at least 4 satellites, Skyhook uses the position estimated using WiFi APs when there are not enough satellites. The root mean square error of Skyhook computed after convergence of the position estimation in the aforementioned environment is around 20.4 m.

The main bottleneck of above approaches is that existing commercial services solve the localization problem *using the different radio chipsets as monolithic entities*. In the following section we instead pose the question of how we can fuse raw measurements from SVs (even with only 1 or 2 SVs in range) with the ones of other radio technologies.

[1] http://www.skyhookwireless.com/products/precision-location

3 FUSING RANGES

In this work, we assume that the client opportunistically exploits any available anchors (GPS satellites and WiFi APs in the study) and infers the ranges to these anchors to compute its position. A high-level representation of the system with ranges from GPS and WiFi technologies is shown in Fig. 3.

We compute ToF ranges using WiFi and GPS technologies. A linear relation holds between the ToF (propagation time) t_{ToF} of radio signals and the distance d, $d = c \cdot t_{\mathrm{ToF}}(d)$, where c indicates the speed of light. Ranges (regardless of the technology) are *computed by the device* that aims to pervasively position itself. ToF ranging requires one way ranging for GPS. We use two-way ranging for WiFi, which has the advantage of being 802.11 standard-compliant [7].

Our client operates as follows:

(a) It receives the GPS signals from satellites (SVs) and sends DATA frames to WiFi APs in communication range.
(b) It estimates the ToF from the SVs and APs.
(c) It computes its position with a multi-lateration multi-technology algorithm. Computation of the position could be also partially offloaded to the cloud.

We use standard methodologies for (a). We then first investigate (b) in Section 3.1 and then (c) in the subsequent sections.

3.1 WiFi ToF shortest path estimator

While measuring ToF ranges to SVs is a well-understood problem, this does not hold for WiFi. WiFi ranging takes advantage of the nominal systematic time bias between the transmission of DATA and the reception of ACK frames. This time offset is due to the fixed 802.11 standard SIFS (Short Interframe Space) time needed by the AP to reply with an ACK after a successful reception of DATA. This means also that the measured ToF is independent from congestion since the ACK is sent systematically after this SIFS. Subsequently, neglecting the effect of noise, any offset from the bias is due to the distance between the client and the AP. The WiFi client does not need to authenticate with the AP. In our tests, DATA sent by the WiFi client are successfully ACKed by APs without being authenticated (most likely because the APs need to schedule the ACK in the fimrware in a very short time, and they do not have the time to verify if the WiFi client is authenticated or not).

For fusion, we need a shortest path estimator that does not require any offline or online environment calibration. In fact, environment calibration may be done with the help of the links between APs in a system where ranges are computed by the APs [8]. Instead, there is no control on the AP in a system where ranges are computed by the mobile device.

In order to address this problem, we consider that, given N measurements, some of them use the Line of Sight (LOS) path (or the shortest Non Line of Sight (NLOS) path in case the LOS path does not exist), and others have one or more NLOS paths. For each single path, we consider a Gaussian distribution for the noise generated by the AP replying with ACKs. Experimental observations of Gaussianity of the single path can be found in [8]. We model the sum of all these multipath components as a *Gaussian Mixture Model (GMM)*. A key aspect of the model is to identify the number of dominant paths (clusters) κ, which is up to 5 in indoor environments [9]. We infer the optimal $\kappa \in \{1, \ldots, 5\}$ for the GMM statistical model

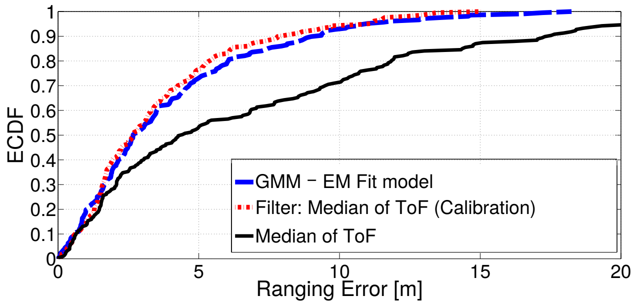

Figure 2: Empirical Cumulative Distribution Function of distance estimation using our WiFi ToF GMM model versus the filter introduced in [8], 207 links.

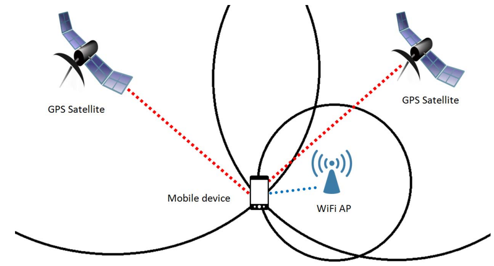

Figure 3: Positioning with ranges from different technologies.

computing the lowest Akaike Information Criterion (AIC) for the N measurements [10]. We then divide the N measurements in k paths with the GMM components likelihood optimized using the iterative Expectation-Maximization (EM) algorithm initialized by k-means++ [11]. We then compute the means of the k paths.

Our estimator rejects the paths with negative means, as they do not correspond to a physical propagation path, and uses the *path with the least positive mean* as input of the measurement model $\mathbf{z_k}$. We use the raw data set from [8] and we consider rounds of 20 measurements to estimate the distance. We show in Fig. 2 how close is the ranging error using the GMM fit model to the filter presented in [8] *without the need of performing any environment calibration.* For this reason, the approach we propose is a step beyond the one proposed in [8] since it brings more flexibility while keeping the accuracy at a relatively similar level.

3.2 Data fuser for positioning

When the total number of ranging measurements of the different technologies is higher than the number of variables to estimate, the location problem can be posed as an optimization problem. We opt for the Extended Kalman filter (EKF) as a data fuser for the raw ranges as it gives us the possibility to treat any measurement differently, taking into account factors related to the technology used for the specific measurement. Using the EKF fuser, we express the positioning problem as a discrete-time process with state model $\mathbf{x_k} = f(\mathbf{x_{k-1}}) + \mathbf{w_{k-1}}$, and measurement model $\mathbf{z_k} = g(\mathbf{x_k}) + \mathbf{v_k}$ where $\mathbf{x_k}$ is the state vector, $\mathbf{z_k}$ the measurement vector, and $\mathbf{w_k} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q_k})$ and $\mathbf{v_k} \sim \mathcal{N}(\mathbf{0}, \mathbf{R_k})$ represent, respectively, the process noise and the measurement noise with autocovariance matrices $\mathbf{Q}$ and $\mathbf{R}$.

3.3 Measurement model

Both GPS and WiFi are affected by a measurement bias:

- GPS bias: it is caused by the clock of the receiver, and it is then client dependent (b_{GPS}). The bias is expressed in meter unit (to have a homogeneous state vector) multiplying the time measurement by the speed of light c. The bias is also subject to the drift d_{GPS} in meter/sec, and thus it must be *estimated continuously* [12].
- WiFi bias: as mentioned in Section 3.1, the WiFi bias is caused by the delay of the receiver to schedule the ACK. Therefore, we have a vector $\mathbf{b}_{\text{WiFi}}$ with size equal to the number of APs in range. This bias is corrupted by large noise [7], but it is not affected by drift. As such, *estimating the bias once is sufficient.*

Using one-way ranging in GPS and two-way ranging in WiFi, we express the measurement function $g(\mathbf{x_k})$ as follows:

$$g(\mathbf{x_k}) = \begin{bmatrix} \{\|\mathbf{p} - \mathbf{p}_{\text{anchor},i}\| + b_{\text{GPS}}\}_{i \in I} \\ \{2 \cdot \|\mathbf{p} - \mathbf{p}_{\text{anchor},j}\| + b_{\text{WiFi},j}\}_{j \in J} \end{bmatrix}, \quad (1)$$

where $\mathbf{p}_{\text{anchor}}$ refers to the coordinates of the position of the anchor considered for ranging measurements, I indicates the set of satellites and J the set of APs. We assume that the APs position can be fetched from an online database (similarly to Skyhook, Google, and other localization systems). The multiplication factor '2' in WiFi in Eq. (1) accounts for the use of two-way ranging.

Model of the autocovariance noise of the measurement. The terms of the measurement autocovariance matrix $\mathbf{R_k}$ corresponding to GPS can be computed considering typical GPS receivers that use the early-minus-late discriminators method [12] for tracking the code delay of the incoming GPS signal. Here, we use the model of the standard deviation for the early-minus-late discriminators described in [12], with raw data collected with our GPS receiver (cf. Sec. 4).

For the terms of the measurement autocovariance matrix $\mathbf{R_k}$ corresponding to WiFi, the noise of the measurement is mainly due to the uncertainty added by the AP replying with ACKs, which can be of several clock cycles [7], and by the presence of multipath [13]. Multipath is handled by our ranging filter proposed in Section 3.1. We assume that the noise in the measurement consists only on the noise *on the shortest path* added by the AP replying with ACKs. Taking into account that the range is estimated using N consecutive ToF values, the components of the measurement covariance of WiFi are equal to $\frac{\sigma^2_{WiFi}}{N}$ $\forall j \in J$.

3.4 Two-phases state model

We propose a two-phases model of the state that takes advantage of the above considerations that the bias of GPS must be estimated continuously while the one of WiFi should be estimated only once. The client first connects to an online database to verify if the AP bias is available. If not, it goes to phase I, where $\mathbf{b}_{\text{WiFi}}$ is unknown and part of the state vector. Here, the client estimates itself the set

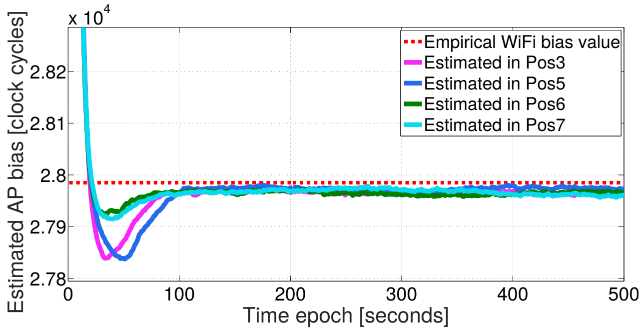

Figure 4: Accuracy of the WiFi bias chipset estimation with state model in phase I.

of WiFi bias $\mathbf{b}_{\mathrm{WiFi}}$. As $\mathbf{b}_{\mathrm{WiFi}}$ depends on the chipset of the AP [7], calibration of each WiFi AP chipset is needed. In fact, every AP may add a bias different from other APs. Once the bias per AP has been estimated, the value is added to the online database. On the other hand, the GPS bias b_{GPS} is added at the level of the GPS receiver as GPS is a one-way Time-of-Flight ranging technology. It is estimated at every time epoch iteratively as a component of the EKF state vector.

Despite there is the very new 802.11mc standard providing the methods to support timing measurements, it is not realistic to suppose that it will be implemented in every AP worldwide in a reasonable time frame. Our method to solve this problem is to focus on scenarios with *only* one AP (AP_j) and enough GPS SVs. For instance, this occurs when the mobile is outdoor with good satellite visibility and AP_j (indoor or outdoor) is in communication range. Under this hypothesis, the state vector is composed by 5 unknowns (x, y, z, b_{GPS} and $b_{\mathrm{WiFi},j}$, where j indicates the specific AP that we have to model) and the EKF filter can be solved using at least 5 ranges (one AP and at least 4 SVs). Once $b_{\mathrm{WiFi},j}$ is estimated (together with the other unknowns), the AP can be used for fused positioning (phase II). In phase II, $\mathbf{b}_{\mathrm{WiFi}}$ is removed from the state vector and a state vector with 4 unknowns (x, y, z, b_{GPS}) is used, as in standalone GPS.

Model of the autocovariance noise of the process. For the autocovariance matrix of the process noise $\mathbf{Q_k}$, we use the kinematic equation of movement for the terms of the state vector related to position and velocity, and similarly for the bias and drift. For the WiFi bias in the process noise $\mathbf{Q_k}$, we use the same fixed standard deviation value σ^2_{WiFi} for each AP.

WiFi bias online estimation. We study the accuracy of phase I of the EKF model, considering all available GPS SVs (without removing any range) for position. We have two observations. First, from Fig. 4, we notice the robustness of the estimator that shows similar performance regardless of the selected position. Second, the value is close (approximately 2 cycles smaller) to the empirical value measured experimentally in controlled experiments.

4 INDOOR EVALUATION SETUP

For the evaluation of the proposed system, unless otherwise stated, we place the client in a total of 10 *indoor* positions as shown in Fig. 5, where it can receive signals from different SVs (yet at low accuracy) and a total of 5 APs. The environment features a rich-multipath experience with several offices and walls. The measurements were collected at different days and time of the day in order to have distinct GPS satellites constellations and indoor conditions. Ground truth evaluated positions and WiFi AP positions were determined in the LLA coordinate system (latitude, longitude and altitude) using an API to integrate the office's building map to Google Earth and thus determine those positions as precisely as possible. The coordinates are further converted to the ECEF (earth-centered, earth-fixed) coordinate system for homogeneity with the GPS satellites coordinates.

Figure 5: Testbed map.

GPS measurements. We collect real traces using the Evaluation Kit with Precision Timing manufactured by U-blox and equipped with an active GPS antenna of type u-blox ANN-MS. Data traces contain pseudoranges as well as other parameters such as satellite clock offset, the ephemeris data broadcasted by the satellites that includes the SV clock bias, drift and drift rate, the Keplerian parameters [12], perturbation parameters, etc.

WiFi measurements. For WiFi ranging, we use ToF two-way ranging based on regular 802.11 DATA frames sent by the transmitter and acknowledged by the receiver with 802.11 ACK frames [14]. The approach that we consider is readily suitable for scenarios where the firmware only provides the number of WiFi clock cycles t_{ToF} between the end of the DATA transmission and the end of the ACK reception. In this work, we use a customized version of the 802.11 openFWWF firmware and b43 driver to perform ranging measurements running on Broadcom AirForce54G 4318 mini PCI type III chipsets. We fix σ_{WiFi} equal to 6 clock cycles in our tests. The clock resolution of the single measurement is equal to the WiFi main clock (88 MHz in this work).

5 CONSTRAINED ENVIRONMENT

The fusion of ranges as performed by the proposed data fuser allows to locate the client in constrained environments where there

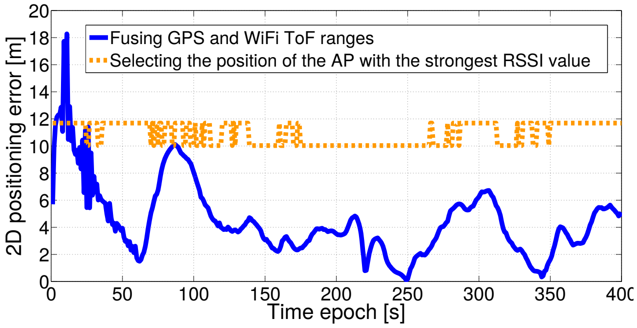

Figure 6: Constrained scenario with few ranges per technology.

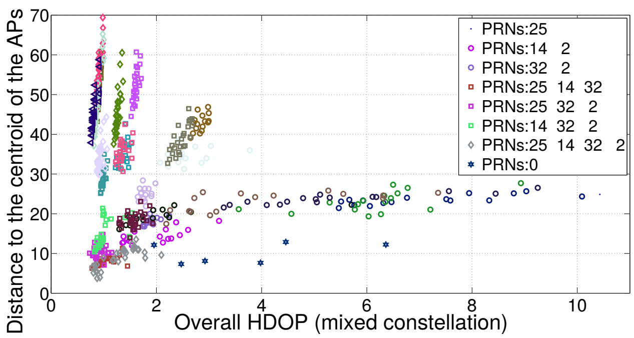

Figure 7: Distance to centroid versus HDOP pair for a specific set of satellites among all the ones in range. The color indicates a specific set of satellites and different values with the same color indicate a different set of APs (PRN (pseudo-random noise number) is a label used to uniquely identify a GPS satellite, "PRNs:0" corresponds here to the pure WiFi case).

are only a few SVs and APs for positioning. For the evaluation, we consider a scenario with only 3 SVs and 2 APs available for positioning. We use only a subset of 2 WiFi APs from the 5 APs deployed in the indoor testbed described in Sec. 4 to understand well how the GPS and WiFi range fusion performs in this specific context. As standalone GPS requires at least four SVs for 3D positioning to find the four unknown variables (x, y, z, b_{GPS}) and standalone WiFi requires at least 3 APs for 2D positioning, none of them is capable to locate the device using a traditional multi-lateration algorithm. The best that can be done with this data set alone is to use the two APs and simply consider the position of the WiFi AP with strongest signal strength as position fix. We show the results in Fig. 6 as a function of time. In the same figure, we also show the accuracy achieved fusing ranges of GPS and WiFi with the EKF introduced in Sec. 3. We observe that our approach gives better accuracy than a monolithic architecture. The RMSE (Root Mean Square Error) of fusing ranges is 3.90 m while selecting the AP with the strongest signal strength gives an error of 10.80 m.

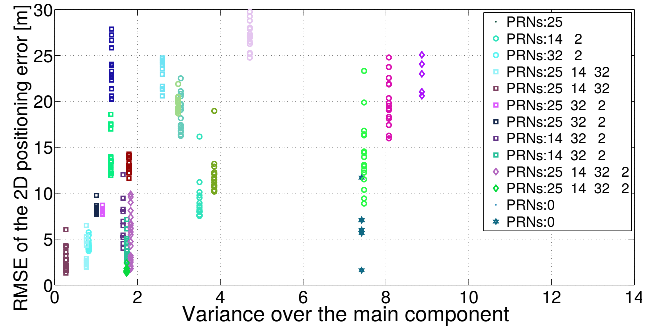

Figure 8: RMSE accuracy as a function of the variance over the principal component after clustering.

6 INDOOR WITH RICH NUMBER OF RANGES

It is also fundamental to study scenarios where there is the availability of a rich set of GPS ranges and WiFi APs, which poses the problem of *which ranges to use* as input to the EKF. Our study is conducted using the indoor scenario presented in Section 4, where some ranges can be largely affected by multipath.

6.1 Geometrical-Statistical approach

We propose to use two geometrical metrics for the selection of the anchors. First, we adopt the centroid of the APs as a reference point with respect to the estimated position of the client $\hat{\mathbf{p}}$. Second, the horizontal dilution of precision (HDOP) of all the anchors. In GPS systems, the HDOP is a term that accounts for the multiplicative effect of the geometry of the satellites on the positioning accuracy [15]. Smaller is the HDOP, smaller is the sensitivity of the position solution with respect to the errors in the ranges. We can use the same formulation for a multi-technology positioning system. In fact, the HDOP only depends on the unit vector of the direction between the receiver and the anchor rather than the coordinates and the distance separating them [15]. It is computed based on the estimated mobile client's position.

We estimate the position in all the locations in the testbed using all possible set of anchors and plot the results in Fig. 7. The color of each result in the figure indicates the (Distance to Centroid, HDOP) pair for a specific set of SVs among all the ones in range. Different values with the same color indicate a different set of APs. We observe that the pairs (Distance to Centroid, HDOP) are clustered based on the set of SVs used for the position fix, alongside with all the combinations considering from one to the maximum number of WiFi APs.

We compute the k-means clustering on the sets of selected SVs with large sizes (corresponding to multiple options for the selection of APs). This allows us to operate with relatively homogeneous sizes. We then perform a principal component analysis (PCA) to compute the variance of the clustered data. Results are shown in Fig. 8. With this approach we are able to choose the best set of SVs "PRNs: 25 14 32". In fact, computing the RMSE of the position errors for a given selection of SVs, we observe that *less spread of the cluster tends to correspond to better accuracy.* The reason is that the geometry of the considered GPS SVs remains robust regardless

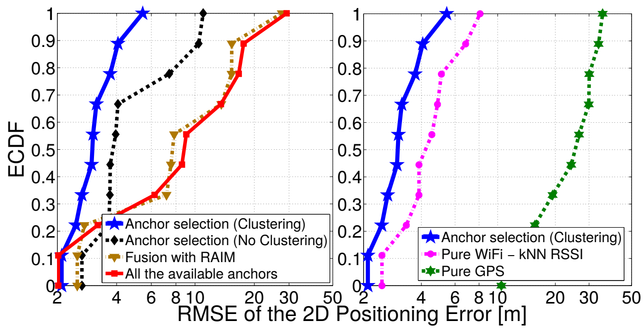

Figure 9: The ECDF of the RMSE (Root Mean Square Error) of the 2D positioning accuracy in meters across different *indoor* positions.

of the set of APs they are fused with. In turn, this indicates that the *selected set of SVs* is relatively reliable. In the final step, *we select the APs* within the selected set of satellites such that the overall HDOP is minimal, which, as stated above, guarantees low sensitivity with respect to small variations in the noise in the ranges. The presented approach could run either on the mobile or could be offloaded to the cloud (that would receive GPS and WiFi range estimations).

6.2 Evaluation of Anchor Selection

We perform an evaluation of our proposed solution for anchor selection. We conduct the evaluation with our system in phase II, using bias values of WiFi APs estimated in phase I (cf. Sec. 3.4). Fig. 9 shows the RMSE positioning error for different single and multi-technologies localization algorithms. On the left side we show a comparison of the different fusion strategies, while on the right we show a comparison of the fusion approach versus standalone WiFi with SNR measurements and standalone GPS estimator. For the SNR-based approach, we use the k-Nearest Neighbor machine learning algorithm (as typical in current WiFi-based location systems [1]).

Simply fusing pseudoranges from these SVs with all the available WiFi ranges is better than relying exclusively on GPS, with an overall accuracy of 11.78 m. As reference methodology, we use the RAIM algorithm that is designed for integrity monitoring [16]. RAIM has the objective of removing one faulty range in case there is redundancy in the number of measurements. However, although some multipath can be detected by RAIM, it is known to fail to detect short delay multipath, which abounds in urban and indoor environments. This is confirmed by our study: trying to reject the non-reliable anchors using RAIM algorithm does not significantly enhance the positioning accuracy, with an average error of 11.07 m. Fig. 9 (left) shows that the statistical-geometrical selection outperforms any other fusion strategy. Using the anchors selection approach with clustering, we achieve an accuracy that is 74.2% higher than applying the anchors selection method without first sub-clustering the sets. From Fig. 9 (right), we can observe that our statistical-geometrical anchor selection algorithm drops down the average RMSE positioning error to 3.29 m, providing a gain of 8*x* with respect to standalone GPS, and better than standalone WiFi positioning with an average of 49.4% accuracy improvement. The latter would be the technology used by current monolithic architectures that do not exploit raw measurements and select the radio providing the best performing localization system.

7 CONCLUSION

We have proposed to fuse timing range measurements of GPS and WiFi technologies, in order to circumvent the limitations of the individual radio access technologies and improve the overall localization accuracy and pervasiveness. We have introduced a data fuser based on a two-phases EKF algorithm and a robust statistical estimator of WiFi ToF ranges. We have shown that fusing ranges is beneficial both in the scenarios where the standalone technology does not have enough anchors to provide a position fix and in the scenarios where there exists a rich set of multiple ranges, yet affected by multipath errors. Our approach is a step beyond classical approaches where technologies in multi-radio access are considered as monolithic entities. It can be extended to other communication technologies that provide timing information such as cellular networks. The computational overhead of the solution we propose with a rich number of ranges needs further investigation and is the plan for our future work.

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